

PROBABILITY – BASIC CONCEPTS

Outline

- Terminology
- Basic Laws of Probability
- Joint and Conditional Probabilities

Terminology

- Usually, probabilities are descriptions of the likelihood of some *event* occurring (ranging from 0 to 1).

$$P(X) = \frac{\text{\# occurrences of } X}{\text{\# of possible occurrences of } X}$$

- The probability of two events occurring will be termed *independent* if knowledge of the occurrence or non-occurrence of the first event provides no effect on estimates of the probability that the second event will occur.

e.g., P(me studying memory) vs. P(you studying memory)

P(Jason (my undergrad) studying memory)

Terminology

- Two events are termed *mutually exclusive* if the occurrence of one event precludes occurrence of the other event.

P(you getting THE car) vs. P(bro getting THE car)

- A set of events are termed *exhaustive* if they embody all of the possible outcomes in some situation.

P(flipping a head) vs. P(flipping a tail)

Basic Laws of Probability

- *The additive law of probabilities:* given a set of mutually exclusive events, the probability of occurrence of one event or another event is equal to the sum of their separate probabilities.
 - Place 100 marbles in a bag; 35 blue, 45 red and 20 yellow.

$$P(\text{blue}) = .35, P(\text{red}) = .45, P(\text{yellow}) = .20$$

- What is the probability of choosing either a red or a yellow marble from the bag?

$$\begin{aligned} P(\text{red or yellow}) &= P(\text{red}) + P(\text{yellow}) \\ &= .45 + .20 \\ &= .65 \end{aligned}$$

Basic Laws of Probability

- *The multiplicative law of probabilities:* The probability of the joint occurrence of two or more independent events is the product of their individual probabilities.
 - Say that the probability that I am in my office at any given moment of the typical day is 0.65.
 - Also, say that the probability that someone is looking for me in my office at any given moment of the school day is 0.15.

Basic Laws of Probability

Example . . . continued:

- What is the probability that that during some particular moment, I am in my office and someone looks for me there?

$$\begin{aligned} &P(\text{in office } \underline{\text{and}} \text{ someone looks}) \\ &= P(\text{in office}) \times P(\text{someone looks}) \\ &= .65 \times .15 \\ &= .0975 \end{aligned}$$

Basic Laws of Probability

- Say that Fred takes the car into work with a probability of .50, walks with a probability of .20, and takes public transit with a probability of .30.
- Barney, on the other hand, drives into work with a probability of .20, walks with a probability of .65, and takes public transit with a probability of .15.
- What is the probability that Fred walked or drove to work and Barney walked or took public transit to work, assuming Fred and Barney's behaviour to be independent?

Joint and Conditional Probabilities

- The *joint probability* of two events A & B is the likelihood that both events will both occur and is denoted as $P(A,B)$.
- When the two events are independent, the joint probability simply follows the multiplicative rule.

Thus, $P(A,B) = P(A) \times P(B)$.

- When they are not independent, it gets a little trickier. . . but we won't worry about that for now.

Joint and Conditional Probabilities

- A *conditional probability* is the probability that some event (A) will occur, given that some other even (B) has occurred.

denoted as $P(A/B)$.

Joint and Conditional Probabilities

– An Example: Drinking & Driving

	Accident	No Accident	Total
Drinking	7	23	30
Not Drinking	6	64	70
Total	13	87	100

$$P(\text{Drinking}) = 30/100 = 0.3000$$

$$P(\text{Accident}) = 13/100 = 0.1300$$

$$\begin{aligned} P(\text{Drinking, Accident}) \\ &= P(\text{Drinking}) \times P(\text{Accident}) \\ &= 0.30 \times 0.13 \\ &= 0.0390 \end{aligned}$$

$$\begin{aligned} P(\text{Drinking} \mid \text{Accident}) \\ &= 7/13 \\ &= 0.5385 \end{aligned}$$

$$\begin{aligned} P(\text{Accident} \mid \text{Drinking}) \\ &= 7/30 \\ &= 0.2333 \end{aligned}$$